Statistical methods in NLP
Introduction

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January 19, 2016

today

- course matters
- analysing numerical data with Python
- basic notions of probability
- simulating random events in Python

overview

overview of the course

analysing numerical data in Python

basics of probability theory

randomness in Python

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why statistics in NLP and linguistics?

- in experimental evaluations:
  - a HMM tagger T_1 is tested on a sample of 1000 words and gets and accuracy rate of 0.92 (92%). **How precise** is this measurement?
  - a Brill tagger T_2 is tested on the same sample and gets an accuracy rate of 0.94. Is the Brill tagger **significantly** better than the HMM tagger?

why statistics in NLP and linguistics?

- in investigations of linguistic data:
  - what is the probability of object fronting in Dutch?
  - do speakers affected by Alzheimer's disease exhibit a **significantly** smaller vocabulary?

why statistics in NLP and linguistics?

- in language processing systems:
  - what is the probability of English *case* being translated to Swedish *kapsel*?
  - . . . of a noun if the previous word was a verb?
- data-driven NLP systems: we specify a general model and tune specific parameters by observing our data

course overview

- theoretical part:
  - probability theory
  - statistical inference
  - statistical methods in experiments
- applications in NLP:
  - classifiers, taggers, parsers, topic models
  - machine translation

course work

- lectures – all in **L308** (except tomorrow)
- lab sessions – in the computer lab **G212** (lab 4)
- always on Tuesdays at 10–12 (except March 15) or Thursdays at 13–15
- teachers:
  - **Richard**: gives most of the lectures and supervises the two introductory computer exercises
  - **Prasanth**: machine translation lecture and assignment
  - **Mehdi**: supervises and grades the other assignments

examination

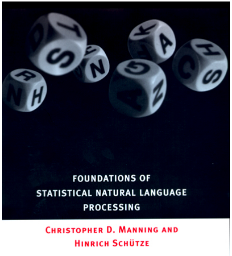
- 2 mandatory computer exercises
- 3 mandatory programming assignments
  - text categorization
  - evaluation
  - PoS tagger implementation
- 2 optional programming assignments for VG
  - topic modeling
  - machine translation

deadlines

- computer exercises: a few days
- programming assignments: 2 weeks after lab session
- VG assignments: March 28

literature

- Manning and Schütze: *Foundations of Statistical Natural Language Processing*



- `http://nlp.stanford.edu/fsnlp/`

literature linked from the web page

- Krenn and Samuelsson: The Linguist's Guide to Statistics – *Don't Panic!*
  - alternative to M&S for the theoretical part of the course
- lecture notes by Michael Collins
  - for the NLP part
  - more in-depth than M&S
- Mitchell: Naïve Bayes and Logistic Regression
- a few other research papers

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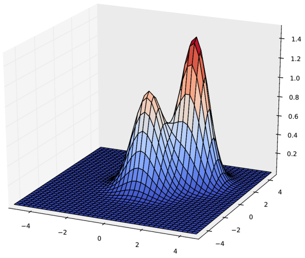
summary statistics and plotting

- if we have some collection of data, it can be useful to summarize the data using plots and high-level measures
- a useful skill in general when carrying out experiments
- and we'll use it in the two computer exercises

some useful Python libraries we'll use in this course

- **SciPy**: a Python library for statistics and math in general
  - http://www.scipy.org/
- **NumPy**: efficient mathematical functions
  - http://www.numpy.org/
- **matplotlib**: using Python to draw diagrams
  - http://matplotlib.org/



an example dataset

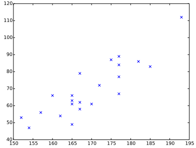
```
177 67 m
177 77 m
175 87 m
154 47 f
157 56 f
152 53 f
165 49 f
165 66 f
165 63 m
162 54 f
167 62 m
167 79 f
167 58 f
165 61 f
```

```
# read the data
data = []
with open('bodies.txt') as f:
    for l in f:
        h, w, s = l.split()
        data.append( (int(h), int(w), s) )

heights = [ h for h, _, _ in data ]
weights = [ w for _, w, _ in data ]

m_heights = [ h for h, _, s in data if s == 'm']
m_weights = [ w for _, w, s in data if s == 'm']
f_heights = [ h for h, _, s in data if s == 'f']
f_weights = [ w for _, w, s in data if s == 'f']
```

plotting the data

- import the plotting library

  `from matplotlib import pyplot as plt`



- plot a height/weight plot, each point as an 'x'

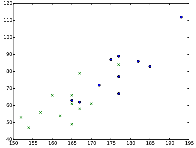
  `plt.plot(heights, weights, 'x')`

- plot height/weight plot by gender

  `plt.plot(m_heights, m_weights, 'o', f_heights, f_weights, 'x')`

- save the plot to a file

  `plt.savefig('myplot.pdf')`



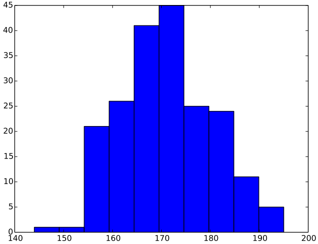
- alternatively, draw the plot on the screen

  `plt.show()`

plotting histograms

- a **histogram** is a diagram that shows how the data points are distributed
- the x axis shows "bins", e.g. 165–170 cm, and the y axis shows the number of data points in that bin
- here's how we draw a histogram with matplotlib:

```
plt.hist(heights, bins=10)
```



some basic data analysis

- **maximal** and **minimal** values:

```
max_f_height = max(f_heights)
print('Tallest female: {0} cm'.format(max_f_height))
```

- **sample mean** (average) and **median**:

```
mean_m_weight = scipy.mean(m_weights)
print('Mean male weight: {0} kgs'.format(mean_m_weight))

median_f_weight = scipy.median(f_weights)
print('Median female weight: {0} kgs'.format(median_f_weight))
```

measures of dispersion: variance and standard deviation

- recall that the mean $\bar{x}$ of a dataset x is defined

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- the **sample variance** $V(x)$ of a dataset x measures how much x is concentrated to the mean
  - it is the mean of the squares of the offsets from the mean

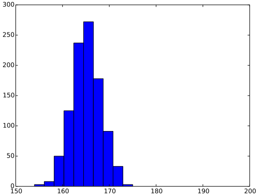
$$V(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- the **sample standard deviation** $\sigma(x)$ is the square root of the variance

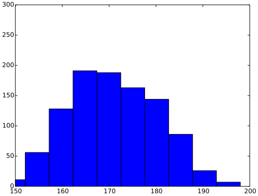
$$\sigma(x) = \sqrt{V(x)}$$

example

- low variance: data concentrated near the mean
  - in the extreme case: all values are identical
- high variance: data spread out



$\sigma = 2.9$



$\sigma = 9.1$

variance and standard deviation in SciPy

- variance:

```
var_height = scipy.var(heights)
```

- standard deviation:

```
std_m_weight = scipy.std(m_weights)
```

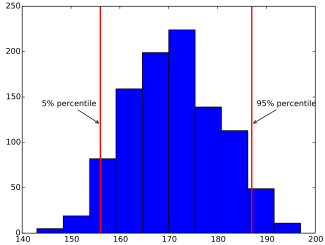
new feature in standard Python: the statistics library

- recently, some statistical functions were added to Python's standard library, for instance
  - `statistics.mean`
  - `statistics.median`
  - `statistics.stdev`
- see https://docs.python.org/3/library/statistics.html

percentiles

- how tall are the shortest 5% of the people in the dataset?
  - formally: what is the x such that 5% of the data is less than x?
- this number is called the **5% percentile**
- in Python:

```
p5 = numpy.percentile(heights, 5)
```



relations between two variables: correlation

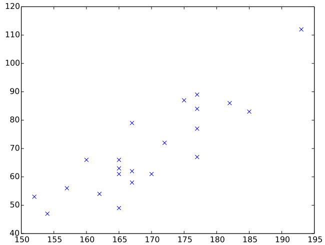
- the **correlation coefficient** or the **Pearson** r measures how close the data is to a linear relationship
- it is a number that ranges between -1 and +1
  - example [Wikipedia]:

- it is defined

$$r(x,y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sigma(x)\sigma(y)}$$

correlation example

- for the height–weight data, $r = 0.87$



- with Python:

```
correlation = scipy.stats.pearsonr(heights, weights)[0]
```

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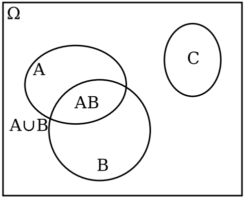
what are probabilities?

- relative frequencies?
  - if we can repeat an experiment: **how often** does the event E occur?
  - when we roll a die, we may say that the probability of a "4" is 1/6 because we will get a "4" approximately 1/6 of the time
- degrees of belief?
  - what is the probability that Elvis Presley is alive?
  - with which probability could Germany have won WW2?

some formal notation: events

- the theory of probability is built on the theory of sets
  - so we can draw Venn diagrams to make the notions more intuitive



- Ω is the **sample set**: the set of all possible situations
- an **event** A is a subset of Ω
- the **union** event $A \cup B$ means that either A or B has happened
- the **joint** event AB (also written $A \cap B$) means that A and B have both happened
- two events A and C that can't happen at the same time (that is, the intersection is empty) are called **disjoint**

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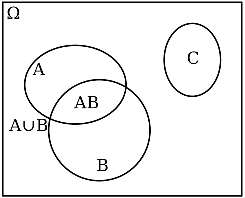
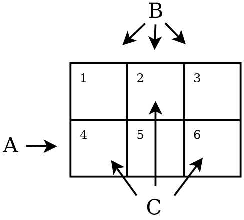
the mathematical definition: the Kolmogorov axioms

- the probability $P(A)$ is a number such that
  - $0 \leq P(A) \leq 1$ for every event A
  - $P(\Omega) = 1$
  - $P(A \cup B) = P(A) + P(B)$ if A and B are disjoint
- in the illustrations, $P(A)$ intuitively corresponds to the area covered by A in the Venn diagram



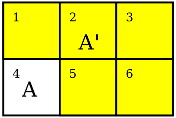
example: Dice rolling

- A = "rolling a 4"; $P(A) = ?$
- B = "rolling 3 or lower"; $P(B) = ?$
- C = "rolling an even number"; $P(C) = ?$
- $P(A \cup B) = P(A) + P(B)$?
- $P(A \cup C) = P(A) + P(C)$?
- P(rolling 1, 2, 3, 4, 5, or 6) $= ?$



some consequences

- $A' =$ "everything but A" $= \Omega \setminus A$



$$P(A') = 1 - P(A)$$

$$P(\emptyset) = 0$$

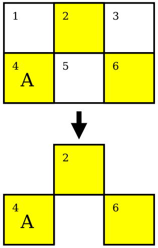
- $A =$ "rolling a 4"; $P(A) = ?$
- $A' =$ "not rolling 4"; $P(A') = ?$
- P(rolling neither 1, 2, 3, 4, 5, 6) $= ?$

joint and conditional probabilities

- the probability of both A and B happening is called the **joint probability**, written $P(AB)$ or $P(A, B)$
- definition: if $P(B) \neq 0$, then

$$P(A|B) = \frac{P(AB)}{P(B)}$$

  is referred to as the **conditional probability of A given B**
- intuitively in the Venn diagram: zoom in on B
  - "what is the probability of a 4 if we know it's an even number?"
- this is something we've already used in language models, taggers, etc



example: vowels in English

- $P(\text{vowel}) = 0.36$
- $P(\text{vowel} \mid \text{previous is vowel}) = 0.12$
- $P(\text{vowel} \mid \text{previous is not vowel}) = 0.50$

the multiplication rule and the chain rule

- if we rearrange the definition of the conditional probability, we get the **multiplication rule**

$$P(AB) = P(A|B) \cdot P(B)$$

- if we have more than two events, this rule can be generalized to the **chain rule**

$$P(ABC) = P(A|BC) \cdot P(B|C) \cdot P(C)$$

- this decomposition is used in taggers, language models, etc

independent events

- definition: two events A and B are **independent** if

$$P(AB) = P(A) \cdot P(B)$$

- this can be rewritten in a more intuitive way: "the probability of A does not depend on anything about B"

$$P(A|B) = P(A)$$

examples

- dice rolling:
  - A = "rolling a 4 the first time"
  - B = "rolling a 4 the second time"
- drawing cards:
  - A = "drawing the ace of spades as the first card"
  - B = "drawing the ace of spades as the second card"

recap: the Markov assumption in language models

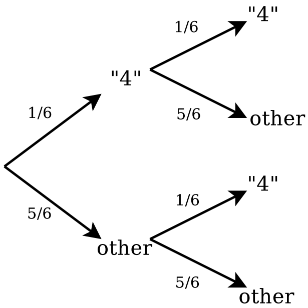
- unigram language model: we assume that the words occur independently

$$\begin{aligned} P(w_1, w_2, w_3) &= P(w_3|w_2, w_1) \cdot P(w_2|w_1) \cdot P(w_1) \\ &= P(w_3) \cdot P(w_2) \cdot P(w_1) \end{aligned}$$

- bigram model: word is independent of everything but the previous word

$$\begin{aligned} P(w_1, w_2, w_3) &= P(w_3|w_2, w_1) \cdot P(w_2|w_1) \cdot P(w_1) \\ &= P(w_3|w_2) \cdot P(w_2|w_1) \cdot P(w_1) \end{aligned}$$

drawing tree diagrams (1)



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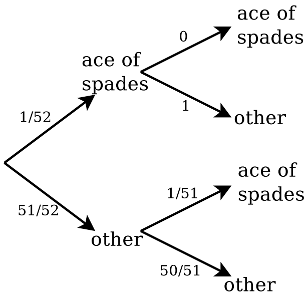
drawing tree diagrams (2)



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a brief note on "random" numbers

- **pseudorandom numbers**: generating "random" numbers in a computer using a deterministic process
  - usually fast
  - we use a starting point called the **seed**
  - if we use the same seed, we'll get the same sequence
    - good for replicable experiments
  - might be a security risk in some situations
- hardware random numbers
  - sample noise from hardware devices
  - Linux: `/dev/random`

basic functions for random numbers: the `random` library

- reset the random number generator

  ```
  random.seed(0)
  ```

- generate a random floating-point number between 0 and 1

  ```
  random_float = random.random()
  ```

- generate a random integer between 1 and 6

  ```
  die_roll = random.randint(1, 6)
  ```

- shuffle the items of a list `lst`

  ```
  random.shuffle(lst)
  ```

- pick a random item from a list `lst`

  ```
  selection = random.choice(lst)
  ```

a note on the random number generators

- the two random number generating functions are examples of **random variables** with **uniform distributions**
  - this means that all outcomes are equally probable
  - if we generate a lot of random numbers, the histogram will be flat
- `random.randint(1, 6)` is a **discrete** uniform random variable
  - it generates 1, 2, 3, 4, 5, or 6 with equal probability $\frac{1}{6}$
- `random.random()` is a **continuous** uniform random variable
  - it generates any float between 0 and 1 with equal probability
- we'll come back to the notion of random variables and their distributions in the next lecture

simulating random events in Python

- random.random and random.randint can be used to simulate random events
- example of generating random words with different probabilities

```
import random

def random_word():
    r = random.random()
    if r < 0.4:
        return 'the'
    if r < 0.7:
        return 'and'
    if r < 0.9:
        return 'in'
    return 'is'

random_words = [ random_word() for _ in range(20) ]

print(random_words)
```

next lecture (Thursday)

- a few more notions from basic probability theory
- random variables and their distributions